

Multi-service Storage Capacity Management by Cumulative Prospect Based Portfolio Theory

Xiaohe Yan, *Member, IEEE*, Yundong Yu, Nian Liu[✉], *Member, IEEE*, Chenghong Gu, *Member, IEEE*, and Furong Li, *Senior Member, IEEE*

Abstract—Energy storage (ES) is vital in electricity markets and its operation is currently forced on single market service. However, ES could get more profits by participating in multiple market services. Thus, the potential of ES is undervalued, and ES cannot get sufficient incentives. In addition, the benefits and risks vary significantly over different markets, which causes severe challenges for ES to manage its capacity optimally over its daily cycle. Portfolio Theory, an optimal trading tool to maximise expected return while minimising corresponding risk, is introduced to quantify the ES capacity allocation over multiple markets. This paper models expected returns and risks of ES over the energy arbitrage market, enhanced frequency response market, and distribution network operator's (DNO's) market. Corresponding to risk over markets, risk aversion level of the storage is evaluated based on the Cumulative Prospect Theory, which is depicted as its indifference curve. The optimum portfolio is calculated by the intersection between the efficient frontier and indifference curve. Then, the ES operation strategy is designed via Robust Optimisation to ensure maximum profit under market price signals uncertainty. Results show the proposed method models risk aversion level of ES more accurately. It also provides an efficient capacity sharing model and daily operation model for ES participating in multiple market services to gain maximum economic return at minimum risk under uncertainty.

Index Terms—Cumulative prospect theory, electricity market, energy storage, portfolio theory, risk aversion.

I. INTRODUCTION

UNDER rollout of a smart and flexible energy system [1], energy storage (ES) capacity is increasing significantly, which required 100 GW by 2020 and 200 GW by 2050 in Western Europe [2]. Although ES is flexible in markets by providing multi-services, it is difficult to allocate ES capacity optimally over markets to increase its profits considering market risks. Currently, the majority of research about ES operation focuses on performance and operational strategies

in a single market [3]–[5], which undervalues ES providing multiple services in electricity systems.

Although some research is starting to explore ES operation in joint markets to increase its profits [6]–[8], it is incomplete without considering risks over markets. Impact of operational policies are evaluated over different services considering degradation and lifespan of ES [6]. A multi-period model and bidding mechanism are applied to operate ES optimally over markets with high renewable penetration [7], [8]. Although ES is applied to maximise profit among energy markets, distribution network operator's (DNO's) markets and balancing markets in [9], risks and uncertainties in these markets are ignored.

With profit increased by participating in multiple markets, ES also should suffer the risk and uncertainty of market prices, simultaneously [10]. There are two key risks for ES in markets, which are 1) operational risk [11] from unpredictable load and ES owners and 2) market risk from commodity price change [12]. In the electricity market, energy price is the key element causing risks for ES due to highly volatility. ES will fail in bidding in markets resulting from these risks. Thus, benefits and risks over markets are significant for ES. Market prices risk are considered with tolerance and prediction error in the operation scheduling method in [13]. Considering uncertainties in forecasting, references [14] and [15] use probability functions and Stackelberg Game theory to schedule ES. However, uncertainty of market prices and the risk aversion level of the ES are not appropriately modeled.

To determine a portfolio of ES based on risk and expected return over multiple markets, several economic models can be implied, such as the index model, arbitrage pricing theory, capital asset pricing model and portfolio theory. Single-index model is applied to qualify risk and the combined effects in farm planning [16]. But it ignores certain factors, which are not accurate to evaluate its outcome. To determine the portfolio, arbitrage pricing theory is applied with return for considering securities and interaction of market factors in [17]. However, it assumes all players to pursue the maximum arbitrage, which ignores economic frictions. Reference [18] discusses a capital asset pricing model with strict assumptions, which assume the mean-covariance matrix is available for all players. Optimal capacity share is determined by portfolio theory based on risk and expected return of ES. But it ignores capacity correlation in a time series over operation cycle [19].

Corresponding to risk in different markets, ES operators allocate their capacity to markets via portfolio theory based on

Manuscript received May 13, 2022; revised August 16, 2022; accepted September 2, 2022. Date of online publication September 8, 2023; date of current version October 18, 2024. This work was supported in part by the National Key R&D Program of China (No. 2021YFB2400701) and the National Natural Science Foundation of China (No. 52107090).

X. H. Yan, Y. D. Yu and N. Liu (corresponding author, email: nianliu@ncepu.edu.cn; ORCID: <https://orcid.org/0000-0001-5971-7995>) are with the State Key Laboratory of Alternate Electrical Power System with Renewable Energy Sources, North China Electric Power University, Beijing 102206, China.

C. H. Gu, and F. R. Li are with the Department of Electronic and Electrical Engineering, University of Bath, U.K.

DOI: 10.17775/CSEEJPES.2022.01520

their risk aversion levels. Risk aversion level is a fundamental parameter determining satisfaction or utility of the return via the utility function [20]. By assuming utility of losses equals that of gain, the Expected Utility Theory is widely applied. References [21], [22] apply this theory to consumer engagement and financial transmission rights to maximise the utility of customers. However, in practice, especially in markets with uncertainty, losses are more sensitive to impact indecision makings from the economic experiment. Thus, Prospect Theory is introduced to assess asymmetric loss and gain perspectives, which reflect different risk aversion levels. Customer behaviour is analysed via prospect theory in reactive power compensation [23] and rationality between voltage and monetary payoffs [24]. However, due to violation of first-order stochastic dominance, prospect theory is improved to Cumulative Prospect Theory [25] by using a probability weighting function derived from rank-dependent expected utility theory. It is more accurate to reflect risk aversion levels of different operators, which is used to solve risk decision-making problems in emergency response [26].

Responding to these key issues for ES participating in different markets under uncertainty, to allocate ES capacity over the energy arbitrage market, Enhanced Frequency Response (EFR) market and DNO's market, portfolio theory is applied. Expected returns and risks of these markets are modeled based on historical data of market prices. Returns in energy arbitrage market is calculated according to energy buying and selling prices. Price in the EFR market is modeled by composition of available price and response price. Congestion cost is the key driver resulting from system operation cost in DNO's market modeling. Based on risk in different markets, the risk aversion degree of the ES is derived from cumulative prospect theory, which represents the indifference curve of the ES. Then, capacity of ES over different markets is allocated by portfolio theory to raise expected return and lower risk. The optimal portfolio of ES at each time is determined by intersection between the efficient frontier and indifference curve. Finally, robust optimisation is proposed to get the optimum in the daily ES operation cycle considering price signal uncertainty in over markets.

This paper is the continuous work of the previous [19], which determines capacity allocation of ES by portfolio theory at a certain time. It cannot guarantee the highest return of ES daily operation. This work integrates portfolio theory with the ES operation strategy by considering the state of charge (SoC) constraint, which extrapolates application of portfolio theory from a single time to the ES operation cycle with time correlation. In addition, the risk aversion level of the ES operators is captured by the cumulative prospect theory and uncertainties of market prices are modeled by a robust set to reduce its impact. Thus, this paper has three key contributions. It i) evaluates risk aversion levels of different ES operators by cumulative prospect theory, which provides a more accurate model to describe economical behaviour of ES; ii) extrapolates application of portfolio theory from a single time to 24 hours considering SoC constraints; iii) it designs a robust optimisation method for ES operators to ensure the highest return in multiple markets with lowest risk during daily

operation under uncertainty of market prices.

The rest of the paper is organized as follows: Section II introduces market models; Section III provides risk aversion level modeling method for ES; Section IV identifies the optimal portfolio by portfolio theory; Section V designs daily robust operation for ES; Section VI demonstrates the proposed method and Section VII concludes the paper.

II. MODELS FOR DIFFERENT ELECTRICITY MARKETS

As a price taker, a third-party owned ES is flexible to choose the markets in which to participate and allocate its capacity over markets. This means the ES does not have the responsibility to provide a certain service and its operation should respond to market prices. Three markets are analysed in this paper: the energy arbitrage market, the DNO's market and the EFR market in ancillary service.

In the energy arbitrage market, ES is charging and discharging during low price periods and high price periods respectively, which ensures expected profits of ES with a big price difference [27], [28]. In the DNO's market, benefit comes from cost savings in investment, operation and maintenance resulting from ES operation. Congestion cost reduction is set in this work considering operation cost in the DNO's market. Distribution network investment could be avoided or deferred by reducing peak power flow, which could be reduced via ES operation during a high load period. In the UK, the EFR market is designed explicitly for ES [29] if they meet the technical requirements of the market. As part of ancillary services in the current commercial frequency market, the EFR market is open to ES above 1 MW with a competitive tender process. It requests ES discharging from maximum operational capacity to minimum operational capacity within 30 minutes [30].

Models of expected return and the risk for these three markets are described in (1)–(2).

$$E_{i,t} = \frac{Pe_{i,t} - Po_{i,t}}{Pb_{i,t}} \quad (i = 1, 2, 3) \quad (1)$$

$$\sigma_{i,t} = \sqrt{\frac{\sum_{x=1}^{X_i} \left(Pe_{i,t,x} - \frac{\sum_{x=1}^{X_i} Po_{i,t,x}}{X_i} \right)^2}{X_i}} \quad (2)$$

where $E_{i,t}$ is the expected return from electricity market, $E_{1,t}$, $E_{2,t}$, $E_{3,t}$ are returns from energy arbitrage, DNO's market and EFR market, respectively. Correspondingly, $\sigma_{1,t}$, $\sigma_{2,t}$, $\sigma_{3,t}$ are risks over these three markets. $E_{1,t}$ is determined by the difference between cost of energy buying ($Po_{1,t}$) and energy selling benefits ($Pe_{1,t}$). Corresponding risk ($\sigma_{1,t}$) is represented by standard deviation of the energy selling prices at each time. $E_{2,t}$ is calculated from the difference between energy buying cost ($Po_{2,t}$) and expected energy selling price ($Pe_{2,t}$). Corresponding risk is standard deviation of congestion amount at each time in the DNO's market. $E_{3,t}$ is calculated from the difference between EFR market profit ($Pe_{3,t}$) and energy cost ($Po_{3,t}$). Corresponding risk is standard deviation of the EFR price at each time. $Pe_{i,t,x}$ is the possible accepted price in X_i number of historical prices in $Pe_{i,t}$ in market i at time t .

Return is determined by congestion cost reduction due to ES charging and discharging, which is zero in a non-congestion system. Congestion cost of the system is shown in (3)–(5).

$$C_{lt} = \frac{TC_t}{(Pw_{l,t} - Pn_{l,t})} \quad (3)$$

$$T_{i,t} = \sum_{l=1}^N \frac{CC_{lt}}{\Delta P_{l,t}} \quad (4)$$

$$\Delta P_{l,t} = AC_PTDF_{ij,l} \times \Delta P_{i,t} \quad (5)$$

where C_{lt} is cost of system congestion from branch l in a N branches system at settlement period t [31], $T_{i,t}$ is cost of system congestion change due to ES operation at node i , $Pw_{l,t}$ is power flow without transmission constraints; $Pn_{l,t}$ is power flow with capacity constraints and TC_t is change of total system energy cost, $\Delta P_{l,t}$ is power flow change on branch l resulting from change of ES charging or discharging ($\Delta P_{i,t}$) in node i , $AC_PTDF_{ij,l}$ is AC Power Transfer Distribution Factors for branch l between node i and j , $T_{i,t,x}$ represents possible congestion cost among the n numbers of branches in $T_{i,t}$ from congestion varying level at this time.

Payment structure of profit in the EFR market normally consists of two fees: response energy fee for operation and availability fee for reserving, substituted in (6). Availability or holding fee (AF) is the available frequency response provider for reserved capacity, given in £/hr in (7). It is a fixed price based on Connection and Use of System Charges [32]. Response energy fee is for operation, which is given to frequency response providers in £/MWh in (8). ES is capable of discharge during low-frequency events and charge during high-frequency events, delivering both responses based on state of charge.

$$REF_t = AF_t + RE_t \quad (6)$$

$$AF_t = (L_t + H_t) \times CA_t \quad (7)$$

$$RE_t = CR_t \times RP_t \quad (8)$$

where L_t is low-frequency £/MW rate and H_t is high-frequency £/MW rate, CA_t is capability of response provider for the time, which is normally in MW; RE_t is Response Energy Fee; CR_t expected response energy for the users. RP_t is reference price which is determined by market index, events number, market index volume and events index.

III. ENERGY STORAGE RISK AVERSION LEVEL EVALUATION

Risk aversion levels vary over different ES operators. They should be evaluated based on their behaviours, which are calculated from their utility function by possessing a superficial generality as the ratio of two derivatives based on the Arrow-Pratt measure of absolute risk aversion [33] in (9).

$$AP = -\frac{V''}{V'} \quad (9)$$

where V is utility function and AP is risk aversion level of ES.

Utility function reflects losses and gains of ES operators according to the cumulative prospect theory [25], [26], which is determined by function pairs π and v in (10). Utility value

of potential outcome is fitted by exponential utility functions in (11).

$$V = \sum_{i=1}^n \pi_i^+ v(x_i) + \sum_{i=1}^n \pi_i^- v(x_i) \quad (i \in n) \quad (10)$$

$$v(x) = \begin{cases} x^\tau & (x \geq 0) \\ -\varphi(-x)^\theta & (x < 0) \end{cases} \quad (11)$$

$$\pi_i^+ = \xi^+ \left(\sum_{j=1}^n h_j \right) - \xi^+ \left(\sum_{j=i+1}^n h_j \right) \quad (12)$$

$$\pi_i^- = \xi^- \left(\sum_{j=1}^i h_j \right) - \xi^- \left(\sum_{j=1}^{i-1} h_j \right) \quad (13)$$

$$\xi^+(h_j) = \frac{(h_j)^\delta}{[(h_j)^\delta + (1 - h_j)^\delta]^{\frac{1}{\delta}}} \quad (14)$$

$$\xi^-(h_j) = \frac{(h_j)^\chi}{[(h_j)^\chi + (1 - h_j)^\chi]^{\frac{1}{\chi}}} \quad (15)$$

where x_i denote values of order statistics for an arbitrary sample with probabilities h_i , $v(x_i)$ is value of potential outcome, π_i^+ is the decision weight for value of potential gain and π_i^- is decision weight for value of the potential loss, τ represents risk aversion respecting to gains and θ represents risk preference in respect to losses and normally $\tau \leq \theta < 1$, ξ^+ and ξ^- denote the weighting functions for gains and losses, χ and δ are parametric values, which define curvature of the weighting function. These parameters capture economic behaviours of the ES, which normally should be determined by surveys.

Utility value of ES can be illustrated in Fig. 1, where the value passes through the reference point is S-shaped and asymmetrical. The reference point on the x -axis serves as the point of separating gains and losses. The utility curve is typically convex for gains and the utility curve is typically concave for losses. Both functions are non-decreasing and take the value of zero at the reference point. It is steeper for losses than gains indicating losses outweigh gains, which means losses are more sensitive to ES operators.

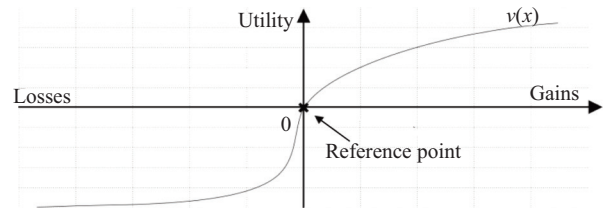


Fig. 1. The utility function.

The probability weighting function in cumulative prospect theory can be described in Fig. 2, which inflates small, and deflates large probabilities. It captures the tendency of storage operators' behaviour when faced with decisions of uncertain outcomes. It is non-decreasing and starts from zero to one, which crosses the 45-degree line and becomes more curved crossing the 45-degree line farther to the right.

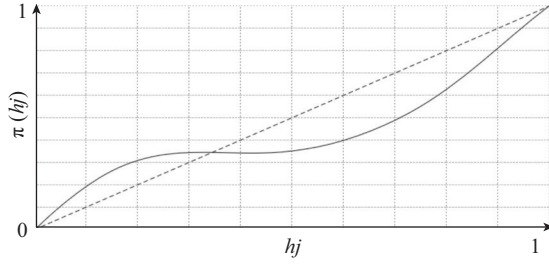


Fig. 2. Probability weighting function.

IV. PORTFOLIO THEORY

Based on risk aversion levels of ES operators, capacity of ES is optimally allocated into markets based on the portfolio theory at each time, which maximises expected return whilst hedging risk correspondingly. It assumes that all ES operators prefer the highest return with the same level of risk and vice versa. ES operators determine optimal portfolios based on expected returns, risks, and correlation of market services.

The portfolio curve is illustrated in Fig. 3 based on the mean-variance theory, which represents ES capacity share changing from 0% to 100% over these three markets. Dots inside the curve is the efficient set or opportunity set, which are possible portfolios of ES over three markets but cannot reach maximum of return at an acceptable risk level. The curve circling the efficient sets (dots) is the efficient frontier, which is the optimal ES portfolio with high expected return and low risk simultaneously. Based on the mean-variance method, efficient portfolios have minimised the risk with a certain return level or maximised return with a certain risk level. The bending degree of the frontier is determined by the coefficient factor between markets and smaller correlation normally leads higher bending degree. Different ES operators have different risk aversion levels, which is illustrated by the indifference curve. For the ES operator with a certain risk aversion level, the optimum portfolio of ES can be calculated based on the intersection point between the efficient frontier and the indifference curve of the operator.

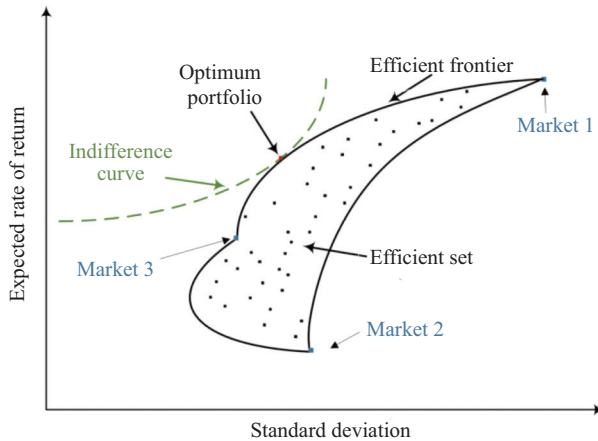


Fig. 3. Efficient frontier and efficient portfolios.

The optimal portfolio [34] for ES with highest return and hedged risk is calculated by utility function (Z) [35] based on

risk aversion level (A) of the ES operator calculated by the cumulate prospect theory:

$$\text{Max: } Z = -\frac{1}{2}AP \times \sigma_p^2 + E_{mp} \quad (16)$$

where E_{mp} is expected return for the portfolio determined by the sum of return in each market with weighted profitability [36]. σ_p is portfolio risk which summarises individual risks and correlation from each and between any two markets.

V. STORAGE OPERATION DESIGN

Under an acceptable risk level, the ES portfolio over markets from each time is extrapolated to their daily operation cycle over 24 hours. The portfolio of ES among different markets should be determined both for its charging and discharging over the daily operation cycle at each time. Thus, the day-ahead operation objective of the ES operator in (17) is to increase its daily expected return over different markets according to their risk aversion level based on the cumulative prospect theory, derived from the portfolio theory in (16) as:

$$\text{Max: } \sum_{t=1}^{24} Z_t = \sum_{t=1}^{24} \left(-\frac{1}{2}AP \times \sigma_{p,t}^2 + E_{mp,t} \right) \quad (17)$$

$$E_{mp,t} = E_{dp,t} + E_{cp,t} \quad (18)$$

$$E_{dp,t} = \sum_{i=1}^n w_{i,t} E_{i,t} P_{d,t} \quad (19)$$

$$E_{cp,t} = \sum_{i=1}^n w_{i,t} (-E_{i,t}) P_{c,t} \quad (20)$$

$$\sigma_p^2 = \sum_{i=1}^n (w_i^2 \sigma_i^2) + \sum_{i=1}^n \sum_{j=1, j \neq i}^n w_i w_j \rho_{ij} \sigma_i \sigma_j \quad (21)$$

$$\rho_{ij} = \frac{M(\sum m_i m_j) - (\sum m_i)(\sum m_j)}{\sqrt{[M \sum m_i^2 - (\sum m_i)^2][M \sum m_j^2 - (\sum m_j)^2]}} \quad (22)$$

$$Em_{i,t} = E_{i,t} + \vartheta_{i,t} \quad (\xi_i \in U_i) \quad (23)$$

$$U_{i,t} = \left\{ E_{i,t} | E_{i,t} = Ep_{i,t} + \vartheta_{i,t}, -1 \leq \vartheta_{i,t} \leq 1, \sum_{i=1}^n \vartheta_{i,t} \leq \Gamma_{lt} \right\} \quad (24)$$

$$0 \leq w_{i,t} \leq 1 \quad (25)$$

$$\sum_{i=1}^n w_{i,t} = 1 \quad (26)$$

$$\text{SoC}_t = \text{SoC}_{t-1} + \frac{(P_{c,t-1} - P_{d,t-1})}{C_{es}} \eta_t \quad (27)$$

$$\text{SoC}_1 = \text{SoC}_1^+ \quad (28)$$

$$\text{SoC}_{\min} \leq \text{SoC}_t \leq \text{SoC}_{\max} \quad (29)$$

$$0 \leq P_{c,t} \leq B \times C_{es} \quad B \in (0, 1) \quad (30)$$

$$0 \leq P_{d,t} \leq (1 - B) \times C_{es} \quad B \in (0, 1) \quad (31)$$

where $E_{mp,t}$ is expected return of ES based on charging or discharging return over markets at each time t , represented

by $E_{dp,t}$ and $E_{cp,t}$, respectively in (18). $w_{i,t}$ is the capacity charge in ES discharging and charging, $P_{c,t}$ and $P_{d,t}$ are charging and discharging amount of ES, $E_{i,t}$ is price for discharging in (19)–(20). $\sigma_{p,t}$ is risk for the portfolio at each time t , σ_i is risk of market share m_i and ρ_{ij} in (22) is the correlation coefficient between costs of the market share m_i and m_j , n is total number of markets in which ES can participate, M is number of data for element s in each market. (23)–(24) are constraints for uncertainty sets of the expected return in different markets. $U_{i,t}$ is uncertainty set of market price, $Ep_{i,t}$ is predicted energy price, $\vartheta_{i,t}$ and Γ_{lt} are conservation levels. The sum of the market share must be equal to '1', represented in constraint (25). The non-negativity condition is shown in constraint (26) for market share proportions. (27)–(29) are equality constraints for state of charge (SoC) and SoC_t is ES SoC, $P_{c,t}$ and $P_{d,t}$ are charging and discharging amount of energy at time t , η_t is efficiency of ES. SoC_1^+ is start SoC of the next daily operation in (28), (29) is non-equality constraints for SoC, (30)–(31) are non-equality constraints for ES charging and discharging, C_{es} is ES capacity, B is a 0, 1 integer, which ensures there is no conflict between charging and discharging process.

VI. CASE STUDY

The proposed ES capacity sharing method is demonstrated in a practical Grid Supply Point (GSP) area test system. in Fig. 4 [37] in the U.K. Energy storage is located on busbar 1007 with a capacity of 12 MW and charging and discharging rate of 3 MW/h, in which efficiency is assumed as 100% in this work. G1 is a PV farm on busbar 1005, with a peak output of 40 MW, supporting domestic demand during daytime. G2, on busbar 1013, is a conventional auxiliary generator and G1008 represents the upstream grid. DC power flow is used to capture active power change during ES operation and network losses are ignored.

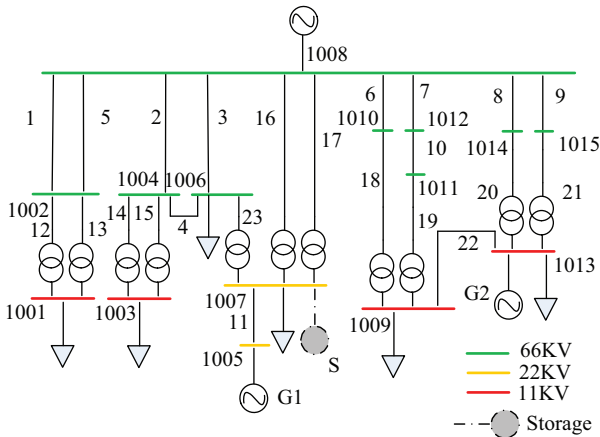


Fig. 4. GSP test system in the UK.

Energy and local generator prices of the system are based on historical data in [19]. The load is supported by local generators when network congestion happens. System energy price is based on energy selling price and local generation

price is determined by operation cost during system congestion periods.

System congestions and branch congestions over the daily ES operation cycle are described in Table I. Resulting from high penetration of PV, branches No.16, No.17 and No.23 have reversed power flow, which are negative value from 12:00 to 13:00. Positive values of congestion, from 16:00 to 22:00, are caused by load. Only these five branches listed in Table I have congestions. Congestions on branches No.16 and No.17 are caused by both generations and load. Highest total congestion is 9.59 MW at 17:00 resulting from the load and generation leads to 7.91 MW congestion at 13:00.

A. The Returns and Risks over Different Markets

Expected return and risk corresponding to ES discharging in the energy arbitrage market, DNO's market and EFR market are listed in Table II. Due to intermittence of congestions, only two markets, the energy arbitrage market and EFR market, are available except for periods from 13:00 to 14:00 and 16:00 to 22:00. Due to inversed power flow resulting from PV output, negative return signals result from system congestions from 13:00 to 14:00 in DNO's market for ES discharging. Correspondingly, risk level during these periods is high due to

TABLE I
THE SYSTEM CONGESTION AND THE BRANCH CONGESTION

	No.2	No.3	No.16	No.17	No.23	Total congestion
12:00	0	0	-0.43	0	-2.01	-2.44
13:00	0	0	-2.64	-1.41	-3.86	-7.91
16:00	0	0	0.82	0	0	0.82
17:00	1.78	3.63	2.71	1.47	0	9.59
18:00	1.44	3.26	2.55	1.32	0	8.58
19:00	0.75	2.54	2.24	1.03	0	6.56
20:00	0	1.46	1.76	0.59	0	3.81
21:00	0	0.75	1.44	0.30	0	2.48
22:00	0	0	0.64	0.00	0	0.64

TABLE II
THE ENERGY PRICE AND EXPECTED RETURNS OVER MARKETS

Time	Energy market		DNOs market		EFR market	
	Return	Risk	Return	Risk	Return	Risk
1:00	0.23	0.39	0.00	0.00	0.39	0.49
2:00	0.17	0.40	0.00	0.00	0.34	0.49
3:00	0.12	0.39	0.00	0.00	0.30	0.48
4:00	0.09	0.38	0.00	0.00	0.28	0.47
5:00	0.03	0.34	0.00	0.00	0.24	0.45
6:00	0.02	0.33	0.00	0.00	0.22	0.44
7:00	0.09	0.38	0.00	0.00	0.28	0.47
8:00	0.20	0.41	0.00	0.00	0.36	0.50
9:00	0.29	0.42	0.00	0.00	0.43	0.50
10:00	0.33	0.41	0.00	0.00	0.47	0.50
11:00	0.39	0.40	0.00	0.00	0.52	0.49
12:00	0.40	0.40	0.00	0.00	0.53	0.49
13:00	0.40	0.40	-3.91	0.88	0.52	0.49
14:00	0.39	0.41	-3.90	0.74	0.51	0.50
15:00	0.34	0.41	0.00	0.00	0.48	0.50
16:00	0.32	0.41	0.00	0.00	0.46	0.50
17:00	0.34	0.41	0.17	0.33	0.47	0.50
18:00	0.38	0.43	2.58	0.55	0.51	0.51
19:00	0.45	0.43	2.38	0.53	0.56	0.52
20:00	0.45	0.43	1.59	0.50	0.56	0.51
21:00	0.40	0.40	0.88	0.45	0.52	0.49
22:00	0.37	0.40	-0.92	0.58	0.50	0.49
23:00	0.34	0.39	-0.06	0.31	0.48	0.49
24:00	0.31	0.39	0.00	0.00	0.45	0.49

uncertainty of renewables. The highest return value is 2.58 in DNO's market, which is much higher than in other markets. Risk is also relatively high in DNO's market. On average, risk and return are low in the energy arbitrage market, where the highest return is 0.45 with a risk level of 0.13. This means more capacity can be allocated to this market if the risk aversion level of ES is low. Availability price is £7/MW/h, which is fixed for ES in the EFR market [6]. Highest return is 0.56 with a risk level of 0.42 in the EFR market, which is higher than in the energy market but lower than in DNO's market.

B. The Risk Aversion Level of Energy Storage

Risk aversion level of ES is determined by its utility based on the cumulated prospect theory. Since the arbitrary sample and behaviour of ES concerning gains and losses should be determined by surveys. This paper use scenarios to show impact of different features in the utility function and probability function in cumulative prospect theory.

Utility of different ESs are the S-shaped curves described in Fig. 5, which corresponding to $\tau = 0.88, \theta = 0.88, \varphi = 2.25$; $\tau = 0.2, \theta = 0.4, \varphi = 1$; $\tau = 0.5, \theta = 0.7, \varphi = 2$, respectively. With larger τ , ES becomes more risk-averse regarding gains. A larger value of θ implies the degree of risk-seeking is more highly respected than losses. With a higher value of φ , the ES is more loss averse.

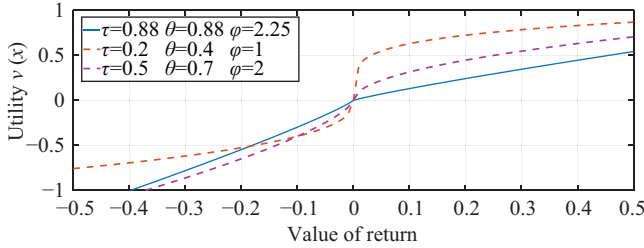


Fig. 5. The utility respecting different returns.

The weighting functions respecting different probabilities are described in Fig. 6 with δ values 0.4, 0.69 and 1. With a smaller value of δ or χ , degree of distortion will be greater in weighting functions.

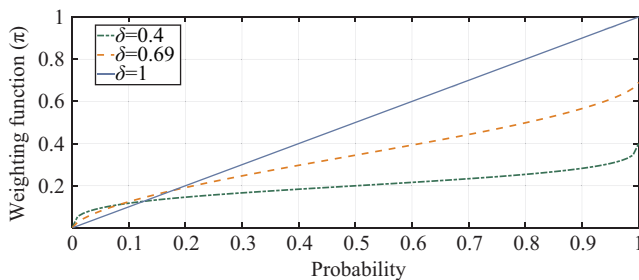


Fig. 6. The Weighting functions respecting different probabilities.

Although utility of the ES is determined by different parameters to describe the economic behaviour of ES, degree of risk aversion level is normally in the range of 2 to 4 derived from deviation of utility. In this paper, risk aversion level is assumed as 3 for the ES.

C. ES Capacity Share at Each Time Point

Based on the risk aversion level of ES, expected return and risk over multiple markets, optimal capacity share is determined by the intersection between the efficient frontier and indifference curve based on portfolio theory. According to the expected return and risk data, capacity of ES allocated to different markets at 16:00 is depicted in Fig. 7. To maximise expected return under acceptable risk, 51% of ES capacity is allocated to the energy arbitrage market and 49% to the EFR market. For this portfolio of ES, expected rate of return is 0.38 and the risk level is 0.21.

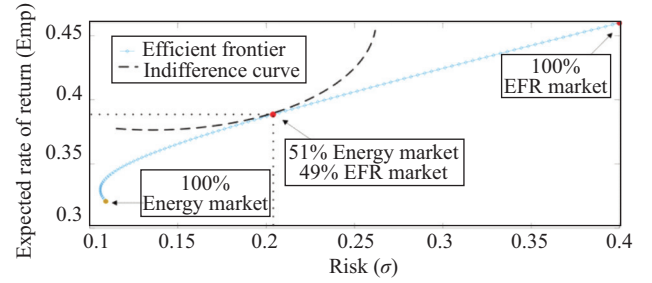


Fig. 7. Optimal capacity share of ES at 16:00.

D. ES Operation in Multi-market

ES operation is to maximise expected return with low risk considering uncertainty from different price signals over multi-markets. Expected return for charging is the negative value and the discharging pricing signals is positive value in Table II.

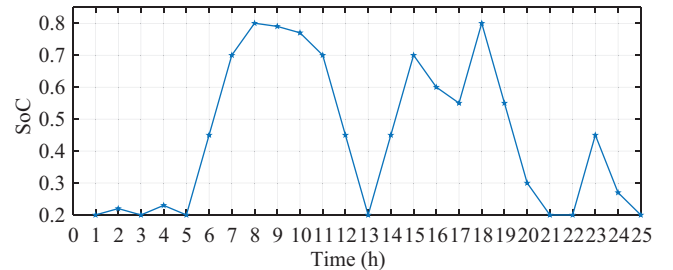


Fig. 8. SoC of ES in the deterministic and robust model.

SoC of the ES is described in Fig. 9. Since maximum and minimum SoC of ES is from 0.2 to 0.8, ES operation should ensure its SoC in this range. By assuming SoC of ES is 0.2 at the beginning of the day, ES is charging when its operation starts. ES is charging from 5:00 to 7:00, 13:00 to 14:00, 17:00 and 22:00, which could get the lowest charging cost for energy buying. SoC reaches maximum at 8:00 and 19:00, which stores energy for discharging in the following hours. ES discharges mainly from 8:00 to 12:00 and 18:00 to 20:00, which is due to high pricing signals from EFR and DNO's market. Maximum charging or discharging rate is 3 MW/h, and SoC should back to 0.2 preparing operation in the next day.

Corresponding to the ES operation scheme in Fig. 9, the optimum portfolio of ES over different markets during 24 hours can be determined and depicted in Fig. 9. The portfolio is described by individual pies with the capacity shares different. The majority of the time, around 60%, ES puts 100% of its

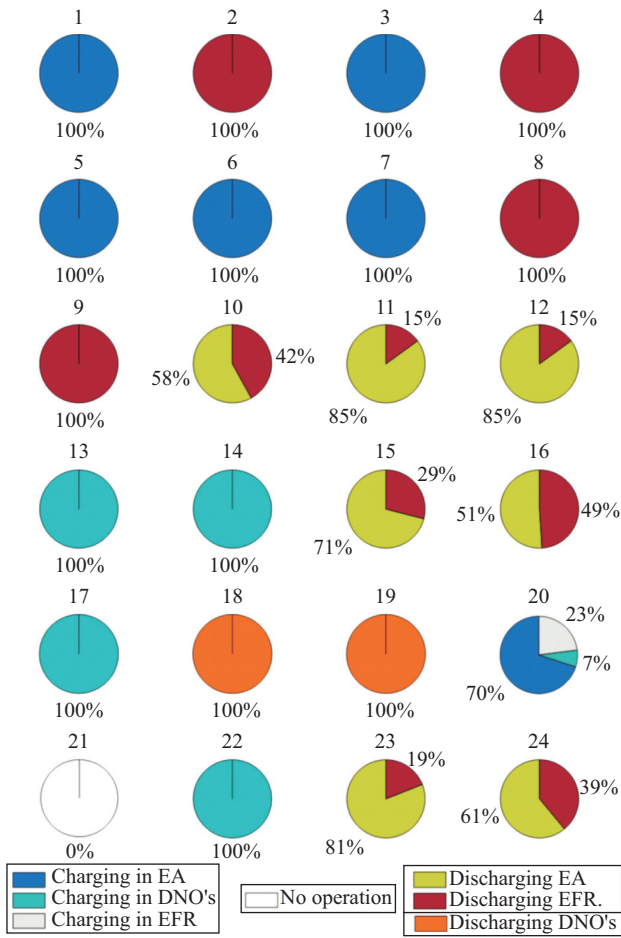


Fig. 9. The portfolios for ES discharging over 24 hours.

capacity in a single market. At 14:00, return is 3.9 for charging in DNO's market, which is 6 times higher than in EFR and risk is 1.3 times higher than in EFR. Thus, 100% of the ES capacity is allocated to the DNO's market. It means return is relatively high in this market, which weakens impact of risk. Except for no operation time at 21:00, the ES participates into two or more markets. Although ES is more willing to put its capacity in the EFR market in discharging due to high return, some capacity should be put in the energy arbitrage market simultaneously to ensure its profits due to high risk in the EFR market. For example, it puts 58% in the EFR market and 42% in the energy arbitrage market at 10:00. Capacity is allocated among three markets at 20:00 during charging time, putting 70% in the energy arbitrage market, 23% in the EFR market and 7% in the DNO's market.

VII. CONCLUSION

This paper designs a novel capacity share method for ES operation over multiple markets considering market risk and expected returns based on portfolio theory during ES daily operation cycle. Key findings are obtained via extensive demonstration:

- Risks aversion levels of different ES operators are more accurately modeled by the cumulative prospect theory.

- Portfolio theory captures risk and expected return of different markets efficiently. The optimum portfolio on the efficient frontier provides more options for ES to gain benefits.
- Robust optimisation-based ES operation method ensures highest return corresponding to its risk aversion level under market price uncertainty.

This work provides a new business model for ES owned by a third party, which is flexible to participate in multiple markets. It unlocks the potential value of ES by dynamically managing its profits and risks over time. Both network operators and the ES owners could get more profits. Future work will focus on ES performing service over multiple markets at the same time.

REFERENCES

- [1] Ofgem. (2016, Nov. 10). A smart, flexible energy system: call for evidence. [Online]. Available: <https://www.gov.uk/government/consultations/call-for-evidence-a-smart-flexible-energy-system>.
- [2] T. E. R. Partnership. (2011). The future role for energy storage in the UK Main Report. [Online]. Available: <http://erpuk.org/project/energy-storage-in-the-uk/>.
- [3] C. Goebel, H. Hesse, M. Schimpe, A. Jossen, and H. A. Jacobsen, "Model-based dispatch strategies for lithium-ion battery energy storage applied to pay-as-bid markets for secondary reserve," *IEEE Transactions on Power Systems*, vol. 32, no. 4, pp. 2724–2734, Jul. 2017, doi: 10.1109/TPWRS.2016.2626392.
- [4] Y. Y. Sun, H. Yue, J. F. Zhang, and C. Booth, "Minimization of residential energy cost considering energy storage system and EV with driving usage probabilities," *IEEE Transactions on Sustainable Energy*, vol. 10, no. 4, pp. 1752–1763, Oct. 2019, doi: 10.1109/TSTE.2018.2870561.
- [5] D. Chen and Z. Jing, "Improved Market Mechanism for Energy Storage Based on Flexible State of Energy," in *CSEE Journal of Power and Energy Systems*, vol. 8, no. 3, pp. 838–848, May 2022, doi: 10.17775/CSEEJPES.2020.02980.
- [6] A. Perez, R. Moreno, R. Moreira, M. Orchard, and G. Strbac, "Effect of battery degradation on multi-service portfolios of energy storage," *IEEE Transactions on Sustainable Energy*, vol. 7, no. 4, pp. 1718–1729, Oct. 2016, doi: 10.1109/TSTE.2016.2589943.
- [7] H. Akhavan-Hejazi and H. Mohsenian-Rad, "Optimal operation of independent storage systems in energy and reserve markets with high wind penetration," *IEEE Transactions on Smart Grid*, vol. 5, no. 2, pp. 1088–1097, Mar. 2014, doi: 10.1109/TSG.2013.2273800.
- [8] P. Zou, Q. X. Chen, Q. Xia, G. N. He, and C. Q. Kang, "Evaluating the contribution of energy storages to support large-scale renewable generation in joint energy and ancillary service markets," *IEEE Transactions on Sustainable Energy*, vol. 7, no. 2, pp. 808–818, Apr. 2016, doi: 10.1109/TSTE.2015.2497283.
- [9] R. Moreno, R. Moreira, and G. Strbac, "A MILP model for optimising multi-service portfolios of distributed energy storage," *Applied Energy*, vol. 137, pp. 554–566, 2015, doi: 10.1016/j.apenergy.2014.08.080.
- [10] J. De Jooode, D. Kingma, M. Lijesen, and V. Shestalova, "Energy policies and risks on energy markets; a cost-benefit analysis," *IDEAS Working Paper Series from RePEc*, 2004.
- [11] U. O. T. E. Industry. (2007). Risk management in the electricity industry white paper I – overall perspective. [Online]. Available: <http://www.eu-relectric.org/media/42893/whitepaperoverallperspectivefinal-2007-030-0137-2-.pdf>.
- [12] P. Sadorsky, "Risk factors in stock returns of Canadian oil and gas companies," *Energy Economics*, vol. 23, no. 1, pp. 17–28, 2001, doi: 10.1016/S0140-9883(00)00072-4.
- [13] M. Kazemi, H. Zareipour, N. Amjadi, W. D. Rosehart, and M. Ehsan, "Operation scheduling of battery storage systems in joint energy and ancillary services markets," *IEEE Transactions on Sustainable Energy*, vol. 8, no. 4, pp. 1726–1735, Oct. 2017.
- [14] Y. Zheng, D. J. Hill, and Z. Y. Dong, "Multi-agent optimal allocation of energy storage systems in distribution systems," *IEEE Transactions on Sustainable Energy*, vol. 8, no. 4, pp. 1715–1725, Oct. 2017.
- [15] Z. Li, L. Chen, W. Wei and S. Mei, "Risk constrained self-scheduling of AA-CAES facility in electricity and heat markets: A distributionally

robust optimization approach,” *CSEE Journal of Power and Energy Systems*, vol. 10, no. 3, pp. 1159–1167, May 2024, 10.17775/CSEEJES.2020.06130.

- [16] R. A. Collins and P. J. Barry, “Risk analysis with single-index portfolio models: an application to farm planning,” *American Journal of Agricultural Economics*, vol. 68, no. 1, pp. 152–161, Feb. 1986, doi: 10.2307/1241660.
- [17] H. Ahmadi, “Testability of the arbitrage pricing theory by neural network,” in *1990 IJCNN International Joint Conference on Neural Networks*, 1990, pp. 385–393.
- [18] Y. C. Shih, S. S. Chen, C. F. Lee, and P. J. Chen, “The evolution of capital asset pricing models,” *Review of Quantitative Finance and Accounting*, vol. 42, no. 3, pp. 415–448, Apr. 2014, doi: 10.1007/s11156-013-0348-x.
- [19] X. H. Yan, C. H. Gu, H. Wyman-Pain, and F. R. Li, “Capacity share optimization for multiservice energy storage management under portfolio theory,” *IEEE Transactions on Industrial Electronics*, vol. 66, no. 2, pp. 1598–1607, Feb. 2018, doi: 10.1109/TIE.2018.2818670.
- [20] P. J. Thomas, “Measuring risk-aversion: The challenge,” *Measurement*, vol. 79, pp. 285–301, 2016, doi: 10.1016/j.measurement.2015.07.056.
- [21] T. G. Papaioannou, V. Hatzi, and I. Koutsopoulos, “Optimal design of serious games for consumer engagement in the smart grid,” *IEEE Transactions on Smart Grid*, vol. 9, no. 2, pp. 1241–1249, 2018, doi: 10.1109/TSG.2016.2582298.
- [22] T. Li and M. Shahidehpour, “Risk-constrained FTR bidding strategy in transmission markets,” *IEEE Transactions on Power Systems*, vol. 20, no. 2, pp. 1014–1021, May 2005, doi: 10.1109/TPWRS.2005.846052.
- [23] Y. P. Wang, W. Saad, A. I. Sarwat, and C. S. Hong, “Reactive power compensation game under prospect-theoretic framing effects,” *IEEE Transactions on Smart Grid*, vol. 9, no. 5, pp. 4181–4193, Sep. 2018, doi: 10.1109/TSG.2017.2652846.
- [24] K. Jhala, B. Natarajan, and A. Pahwa, “Prospect theory-based active consumer behavior under variable electricity pricing,” *IEEE Transactions on Smart Grid*, vol. 10, no. 3, pp. 2809–2819, May 2019, doi: 10.1109/TSG.2018.2810819.
- [25] A. Tversky and D. Kahneman, “Advances in prospect theory: Cumulative representation of uncertainty,” *Journal of Risk and Uncertainty*, vol. 5, no. 4, pp. 297–323, Oct. 1992, doi: 10.1007/BF00122574.
- [26] Y. Liu, Z. P. Fan, and Y. Zhang, “Risk decision analysis in emergency response: A method based on cumulative prospect theory,” *Computers & Operations Research*, vol. 42, pp. 75–82, Feb. 2014, doi: 10.1016/j.cor.2012.08.008.
- [27] H. Khani, R. K. Varma, M. R. D. Zadeh, and A. H. Hajimiragha, “A real-time multistep optimization-based model for scheduling of storage-based large-scale electricity consumers in a wholesale market,” *IEEE Transactions on Sustainable Energy*, vol. 8, no. 2, pp. 836–845, Apr. 2017.
- [28] T. B. C. Group. (2011). Revisiting Energy Storage. [Online]. Available: <https://www.bcg.com/documents/file72092.pdf>.
- [29] N. Grid. (2017). Enhanced Frequency Response. [Online]. Available: <http://www2.nationalgrid.com/Enhanced-Frequency-Response.aspx>.
- [30] Y. Zhou, M. Cheng, and J. Z. Wu, “Enhanced frequency response from industrial heating loads for electric power systems,” *IEEE Transactions on Industrial Informatics*, vol. 15, no. 6, pp. 3388–3399, Jun. 2019, doi: 10.1109/TII.2018.2879907.
- [31] J. T. Li and F. R. Li, “A congestion index considering the characteristics of generators & networks,” in *2012 47th International Universities Power Engineering Conference (UPEC)*, 2012, pp. 1–6.
- [32] N. Grid. (2017). The Connection Use of System Code (CUSC). [Online]. Available: www2.nationalgrid.com/.
- [33] J. Meyer, “Chapter 3 - the theory of risk and risk aversion,” *Handbook of the Economics of Risk and Uncertainty*, vol. 1, pp. 99–133, 2014.
- [34] F. Gökgöz and M. E. Atmaca, “Financial optimization in the Turkish electricity market: Markowitz’s mean-variance approach,” *Renewable and Sustainable Energy Reviews*, vol. 16, no. 1, pp. 357–368, Jan. 2012, doi: 10.1016/j.rser.2011.06.018.
- [35] R. C. Garcia, V. González, J. Contreras, and J. E. S. C. Custodio, “Applying modern portfolio theory for a dynamic energy portfolio allocation in electricity markets,” *Electric Power Systems Research*, vol. 150, pp. 11–23, Sep. 2017, doi: 10.1016/j.epsr.2017.04.026.
- [36] S. Hubbert, *Essential Mathematics for Market Risk Management*. Chichester: Chichester: John Wiley & Sons Ltd, 2012.
- [37] C. H. Gu, F. R. Li, and Y. X. He, “Enhanced long-run incremental cost pricing considering the impact of network contingencies,” *IEEE Transactions on Power Systems*, vol. 27, no. 1, pp. 344–352, Feb. 2012.



Xiaohe Yan received M.S. and Ph.D. degrees from the University of Bath, Bath, U.K., in 2015 and 2019, respectively. From 2019 to 2020, he was a postdoctoral fellowship with University of Macau. He is currently an Associate Professor with the Department of Electronic and Electrical Engineering, North China Electric Power University, Beijing, China. His research interests mainly include energy storage operation, power system planning, analysis, and power system economics.



Yundong Yu received the M.S. degree in Electrical Engineering in 2023 from North China Electric Power University, Beijing, China. His research interests include energy storage operation, power system planning.



Nian Liu received the B.S. and M.S. degrees in Electric Engineering from Xiangtan University, Hunan, China, in 2003 and 2006, respectively, and the Ph.D. degree in Electrical Engineering from North China Electric Power University, Beijing, China, in 2009. He is currently a Professor and the Vice Dean with the School of Electrical and Electronic Engineering, North China Electric Power University. He is also the Director of the Research Section for Multiinformation Fusion and Integrated Energy System Optimization, the State Key Laboratory of Alternate Electrical Power System with Renewable Energy Sources. He is a Member of the Standardization Committee of Power Supply and Consumption, Power Industry of China. He was the Highly Cited Chinese Researcher of Elsevier in 2020 and 2021. From 2015 to 2016, he was a Visiting Research Fellow with the Royal Melbourne Institute of Technology University, Melbourne, VIC, Australia. He has authored or coauthored more than 180 journal and conference publications and has been granted more than 20 patents of China. His research interests mainly include multienergy system integration, microgrids, cyber-physical energy system and renewable energy integration.



Chenghong Gu received the B.S. degree in Electrical Engineering from the Shanghai University of Electric Power, Shanghai, China, in 2003, the M.S. degree in Electrical Engineering from Shanghai Jiao Tong University, Shanghai, in 2007, and the Ph.D. degree from the University of Bath, U.K., where he is currently a Reader with the Department of Electronic and Electrical Engineering. His major research interest is in multi-vector energy systems, resilience, and power economics.



Furong Li received the B.Eng. degree in Electrical Engineering from Hohai University, Nanjing, China, in 1990, and the Ph.D. degree from Liverpool John Moores University, Liverpool, U.K., in 1997. She is a Professor with the University of Bath. Her major research interest is in the area of power system planning, analysis, and power system economics.